REPORT No. 228

A STUDY OF THE EFFECT OF A DIVING START ON AIRPLANE SPEED

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SUMMARY

Equations for instantaneous velocity and distance flown are derived for an airplane which crosses the starting line of a speed course at a speed higher than that which can normally be maintained in horizontal flight. A specific case is assumed and calculations made for five initial velocities. Curves of velocity, average velocity, and distance flown are plotted against time for each case and analyzed. It is shown that the increase in average velocity due to a diving start may be very large for short-speed courses.

INTRODUCTION

In attempts to establish airplane speed records when the method of approach to the speed course is not specified, pilots often dive in order to enter the course at a speed greater than that which can normally be maintained in horizontal flight. The flight over the course is then made at a speed which asymptotically approaches the normal horizontal speed as the excess kinetic energy is absorbed. The increase in average speed thus obtained for courses of varying length should be of considerable interest to pilots and to the officials in charge of contests.

So far as the writer has been able to ascertain, no analysis of this problem has previously been made. In the present analysis, assumptions have been made so as to simplify the problem as much as practicable without seriously affecting the validity of the final results.

ASSUMPTIONS

In order to obtain a simple and reasonably exact solution of the problem the following assumptions have been made:

(1) Propeller thrust is constant,

(2) Flight over the course is horizontal,

(3) The resistance varies as V^2.

The first assumption is a simplifying approximation only. If the brake horsepower and propeller efficiency were to remain constant then the thrust must vary inversely as the velocity. Actually the engine speeds up and delivers an appreciable increase in power when the flight speed is increased, while the propeller efficiency remains substantially constant. The net result is a thrust which neither remains constant nor varies inversely as the velocity. Since the assumption of constant thrust is simpler than that of variable thrust, it has been adopted.

The second and third assumptions are fully justified. One of the requirements always made in speed runs is horizontal or substantially horizontal flight. The change in angle of attack required to maintain horizontal flight is very small under the conditions assumed. Consequently the drag coefficient will be constant and the drag will vary as V^2.

DERIVATION OF EQUATION FOR VELOCITY

The horizontal forces acting on the airplane are thrust and resistance. The equation of motion is

$$F=(T-R)=\frac{W}{g}\frac{dV}{dt} \qquad (1)$$

R may be replaced by its equivalent KV^2, the value of K being taken for R in pounds and V in feet per second, in order to obtain consistent units. Substituting KV^2 for R and rearranging equation (1) gives

$$\frac{dV}{T-KV^2}=\frac{g}{W}dt \tag{1a}$$

which upon integration becomes

$$\frac{2g\sqrt{TK}}{W}t=\log_e\frac{(T-V_o\sqrt{TK})+V(\sqrt{TK}-KV_o)}{(T+V_o\sqrt{TK})-V(\sqrt{TK}+KV_o)} \tag{2}$$

or

$$e^{\frac{2g\sqrt{TK}}{W}t}=\frac{(T-V_o\sqrt{TK})+V(\sqrt{TK}-KV_o)}{(T+V_o\sqrt{TK})-V(\sqrt{TK}+KV_o)} \tag{2a}$$

from which

$$V=\frac{(T+V_o\sqrt{TK})e^{\frac{2g\sqrt{TK}}{W}t}+(V_o\sqrt{TK}-T)}{(\sqrt{TK}+KV_o)e^{\frac{2g\sqrt{TK}}{W}t}+(\sqrt{TK}-KV_o)} \tag{3}$$

In these equations T is the thrust in pounds, t the time in seconds measured from the time of crossing the starting or base line, V_o the velocity in feet per second when $t=0$, V the instantaneous velocity in feet per second, and K the resistance coefficient previously defined.

For simplicity equation (3) may be written in the form

$$V=\frac{C_1e^{at}+C_2}{C_3e^{at}+C_4} \tag{3a}$$

where

$$C_1=(T+V_o\sqrt{TK})$$
$$C_2=(V_o\sqrt{TK}-T)$$
$$C_3=(\sqrt{TK}+KV_o)$$
$$C_4=(\sqrt{TK}-KV_o)$$
$$a=\frac{2g\sqrt{TK}}{W}$$

DERIVATION OF EQUATION FOR DISTANCE

The distance flown in a given time may readily be obtained by integrating equation (3a).

$$S=\int Vdt=\int_{t_o}^{t_1}\left[\frac{C_1e^{at}+C_2}{C_3e^{at}+C_4}\right]dt \tag{4}$$

$$S=\frac{C_1}{aC_3}\log_e(C_3e^{at}+C_4)+\frac{C_2}{aC_4}[at-\log_e(C_3e^{at}+C_4)+C. \tag{5}$$

Equation (5) may now be very much simplified by returning to the original terms, since

$$\frac{C_1}{aC_3}=\frac{(T+V_o\sqrt{TK})}{\frac{2g\sqrt{TK}}{W}\left(\sqrt{TK}+KV_o\right)}=+\frac{W}{2gK} \tag{6}$$

$$\frac{C_2}{aC_4}=\frac{V_o\sqrt{TK}-T)}{\frac{2g\sqrt{TK}}{W}\left(\sqrt{TK}-KV_o\right)}=-\frac{W}{2gK} \tag{7}$$

$$\frac{C_2}{C_4}=\frac{(V_o\sqrt{TK}-T)}{(\sqrt{TK}-KV_o)}=-\sqrt{\frac{T}{K}}=-V_a \tag{8}$$

substituting (6), (7) and (8) into (5) gives

$$S=\frac{W}{gK}\log_e(C_3e^{at}+C_4)-V_0t+C \tag{5a}$$

When $t=0$, $S=0$. Therefore

$$C=-\frac{W}{gK}\log_e(C_3+C_4)$$

$$=-\frac{W}{gK}\log_e(2\sqrt{TK})$$

from which

$$S=\frac{W}{gK}\log_e(C_3e^{at}+C_4)-V_0t-\frac{W}{gK}\log_e(2\sqrt{TK}) \tag{9}$$

APPLICATION OF EQUATIONS TO A SPECIFIC PROBLEM

In order to study the effects of a diving start, a fictitious airplane having characteristics similar to the recent racing designs will be assumed:

Let $W=2{,}100$ lbs.

$V=250$ M. P. H. $=366.67$ f. p. s.

and $T=R=600$ lbs.

Then $K=600/(366.67)^2=.0044628$

$$\sqrt{K}=.0668$$

$$\sqrt{T}=24.4949$$

$$\sqrt{TK}=1.63636$$

$$\frac{W}{gK}=14615.335$$

The equations for velocity and distance may now be written for any initial velocity V_o. Table I contains the evaluation of the constants for five values of V_o: 260, 270, 280, 290, and 300 miles per hour. The resulting equations are:

I. $V_o=260$ M. P. H. $=381.333$ f. p. s.

Velocity
$$V=\frac{1224\,e^{.05014t}+24}{3.33818\,e^{.05014t}-.06545} \tag{10}$$

Distance flown
$$S=14615.34\log_e(3.33818\,e^{.05014t}-.06545)-366.67\,t-17328.30 \tag{11}$$

II. $V_o=270$ M. P. H. $=396.00$ f. p. s.

$$V=\frac{1248\,e^{.05014t}+48}{3.40364\,e^{.05014t}-.13091} \tag{12}$$

$$S=14615.34\log_e(3.40364\,e^{.05014t}-.13091)-366.67\,t-17328.30 \tag{13}$$

III. $V_o=280$ M. P. H. $=410.66$ f. p. s.

$$V=\frac{1272\,e^{.05014t}+72}{3.46909\,e^{.05014t}-.19636} \tag{14}$$

$$S=14615.34\log_e(3.46909\,e^{.05014t}-.19636)-366.67\,t-17328.30 \tag{15}$$

IV. $V_o = 290$ M. P. H. $= 425.33$ f. p. s.

$$V = \frac{1296\ e^{.050141t} + 96}{3.53455\ e^{.050141t} - .26182} \tag{16}$$

$$S = 14615.34 \log_e (3.53455\ e^{.050141t} - .26182) - 366.67\ t - 17328.30 \tag{17}$$

V. $V_o = 300$ M. P. H $= 440$ f. p. s.

$$V = \frac{1320\ e^{.050141t} + 120}{3.60000\ e^{.050141t} - .32727} \tag{18}$$

$$S = 14615.34 \log_e (3.6000\ e^{.050141t} - .32727) - 366.67\ t - 17328.30 \tag{19}$$

Velocities, distances, and average velocities have been calculated from equations (10) to (19), inclusive, and are given in Tables II and III and plotted in Figures 1 to 5, inclusive. A summary of these data is given in Table IV and plotted in Figure 6.

CONCLUSIONS

From a study of the figures and the summary in Table IV, the following conclusions may be drawn:

1. The effect of a dive before crossing the starting line is to increase the average velocity over the speed course by an amount which is directly proportional to the increase in initial velocity relative to the normal horizontal velocity.
2. A 10 per cent increase in initial velocity gives an increase in average velocity of 7.1 per cent over a 1-mile course, 5.2 per cent over a 2-mile course, 4 per cent over a 3-mile course, and 3.1 per cent over a 4-mile course for the specific case investigated.
3. The effect of an increase in initial velocity persists for a longer time than would be expected. At the end of one minute the velocity is still appreciably above normal.
4. Speed records made over courses of different lengths are not comparable when a diving start is taken.

TABLE I

EVALUATION OF CONSTANTS IN THE EQUATIONS FOR VELOCITY AND DISTANCE FLOWN

$W = 2{,}100$ lbs. $V = 250$ M. P. H. $R = 600$ lbs.

V_0 M. P. H.	260	270	280	290	300
V_0 f. p. s.	381.33	396.0	410.66	425.33	440.0
$V_0\sqrt{TK}$	624.0000	648.0000	672.0000	696.0000	720.0000
$C_1 = (T + V_0\sqrt{TK})$	1,224.00	1,248.00	1,272.00	1,296.00	1,320.00
$C_2 = (V_0\sqrt{TK} - T)$	24.00	48.00	72.00	96.00	120.00
KV_0	1.70181818	1.76727272	1.83272727	1.89818181	1.96363636
$C_3 = (\sqrt{TK} + KV_0)$	3.33818181	3.40363636	3.46909090	3.53454545	3.60000000
$C_4 = (\sqrt{TK} - KV_0)$	−.0654545	−.130909	−.1963636	−.261818	−.3272727
$a = \frac{2g\sqrt{TK}}{W}$	.050141	.050141	.050141	.050141	.050141

$T = 600$ $\sqrt{T} = 24.4049$
$K = .0044628$ $\sqrt{K} = .06680$
$\sqrt{TK} = 1.636363$

TABLE II

CALCULATED VELOCITIES

W=2,100 lbs. V=250 M. P. H. R=600 lbs.

Time t sec.	Velocities—M. P. H.				
	V_1=260	V_1=27C	V_1=280	V_1=290	V_1=300
0	260.00	270.00	280.00	290.00	300.00
2	259.03	268.02	276.98	285.92	294.80
4	258.15	266.24	274.28	262.27	290.18
6	257.37	264.65	271.86	279.01	286.07
8	256.65	263.22	269.69	276.10	282.41
10	256.01	261.93	267.74	273.49	278.89
15	254.67	259.23	263.70	268.10	272.39
20	253.62	257.15	260.60	263.97	267.25
25	252.81	255.55	258.21	260.81	263.32
30	252.19	254.31	256.37	258.37	260.30
40	251.32	252.60	253.83	255.04	256.19
50	250.80	251.57	252.32	253.04	253.42
60	250.49	250.95	251.40	251.84	252.25

TABLE III

CALCULATED DISTANCES AND AVERAGE VELOCITIES

W=2,100 lbs. V=250 M. P. H. R=600 lbs.

Time t sec.	V_o=260		V_o=270		V_o=280		V_o=290		V_o=300	
	Distance flown	Average velocity	Distance flown	Average velocity	Distance flown	Average velocity	Distance flown	Average velocity	Distance flown	Average velocity
	Miles	M. P. H.	*Miles*	M. P. H.	*Miles*	M. P. H.	*Miles*	M. P. H.	*Miles*	M. P. H.
0	0	260.00	0	270.00	0	280.00	0	290.00	0	300.00
2	.144	259.23	.149	268.71	.155	278.18	.160	257.46	.165	297.05
4	.288	258.70	.298	267.71	.307	276.68	.317	285.55	.327	294.50
6	.431	258.38	.446	266.95	.459	275.46	.473	263.90	.487	292.38
8	.573	257.84	.591	266.00	.609	274.10	.627	282.12	.645	290.13
10	.715	257.51	.737	265.28	.758	272.98	.780	280.60	.801	268.22
15	1.069	256.64	1.098	263.56	1.127	270.41	1.155	277.15	1.183	283.88
20	1.422	255.93	1.456	262.12	1.490	268.23	1.524	274.26	1.557	260.24
25	1.773	255.31	1.812	260.88	1.850	266.38	1.888	271.80	1.925	277.16
30	2.123	254.78	2.165	259.84	2.207	264.67	2.249	269.67	2.288	274.54
40	2.822	253.75	2.868	258.13	2.914	262.26	2.959	266.32	3.004	270.33
50	3.518	253.28	3.567	256.84	3.616	260.35	3.664	263.78	3.714	267.42
60	4.213	252.79	4.364	255.85	4.315	258.87	4.364	261.88	4.412	264.74

TABLE IV

SUMMARY OF CALCULATIONS

Length of course						
	Initial velocity M. P. H.	260	270	280	290	300
	Ratio $\left\{\frac{\text{initial velocity}}{\text{normal velocity}}\right\}$	1.04	1.08	1.12	1.16	1.20
1 mile	Time, seconds	14.02	13.64	13.28	12.94	12.61
	Final velocity, M. P. H.	254.8	259.8	265.1	270.0	275.1
	Average velocity, M. P. H.	256.8	264.0	271.3	278.4	285.8
	Ratio $\left\{\frac{\text{average velocity}}{\text{normal velocity}}\right\}$	1.027	1.056	1.085	1.113	1.143
2 miles	Time, seconds	28.25	27.66	27.10	26.56	26.03
	Final velocity, M. P. H.	252.4	254.8	257.3	260.1	262.6
	Average velocity, M. P. H.	255.0	260.3	265.6	270.5	276.6
	Ratio $\left\{\frac{\text{average velocity}}{\text{normal velocity}}\right\}$	1.020	1.041	1.062	1.082	1.106
3 miles	Time, seconds	42.56	41.88	41.22	40.58	39.95
	Final velocity, M. P. H.	251.2	252.5	253.7	255.0	256.2
	Average velocity, M. P. H.	253.8	257.9	262.0	266.1	270.4
	Ratio $\left\{\frac{\text{average velocity}}{\text{normal velocity}}\right\}$	1.015	1.032	1.048	1.064	1.082
4 miles	Time, seconds	56.93	56.21	55.50	54.80	54.09
	Final velocity, M. P. H.	250.6	251.2	251.7	252.3	252.8
	Average velocity, M. P. H.	252.9	256.2	259.5	262.7	266.2
	Ratio $\left\{\frac{\text{average velocity}}{\text{normal velocity}}\right\}$	1.012	1.025	1.038	1.051	1.065

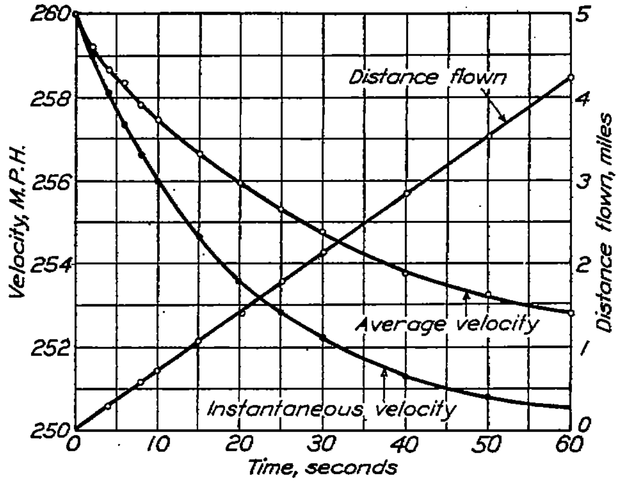

FIG 1.—Initial velocity 260 M. P. H.

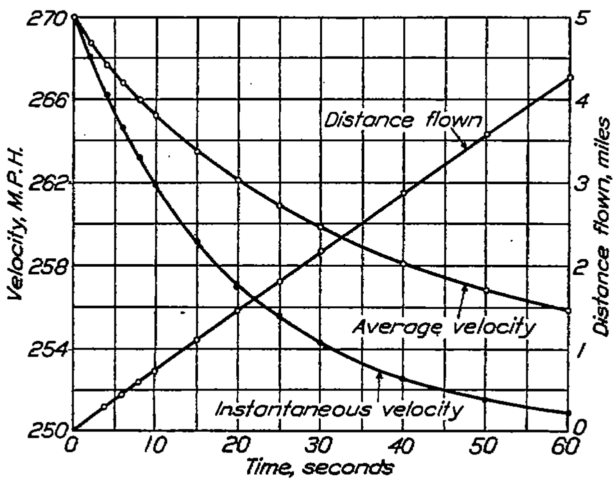

FIG. 2.—Initial velocity 270 M. P. H.

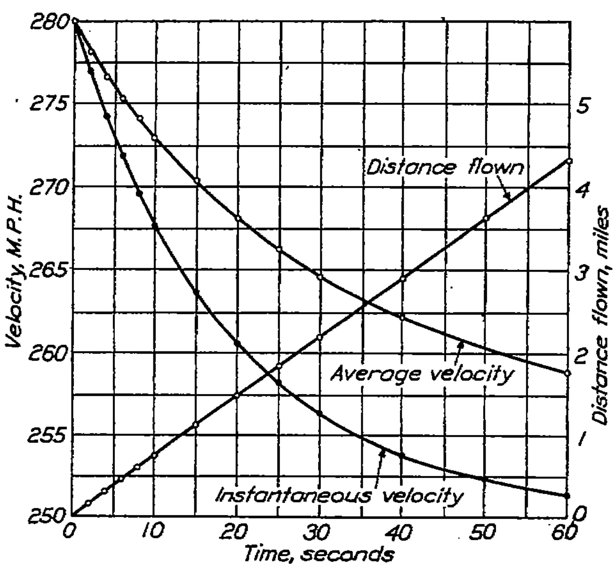

FIG. 3.—Initial velocity 280 M. P. H.

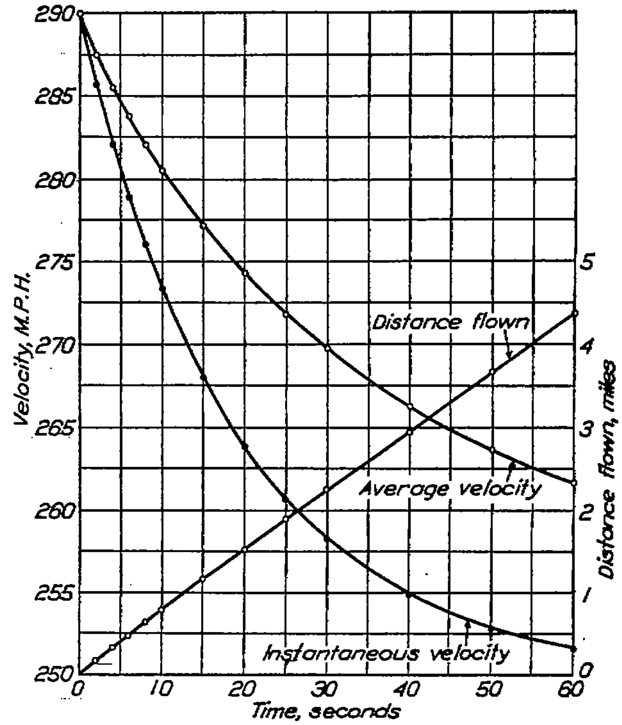

FIG. 4.—Initial velocity 200 M. P. H.

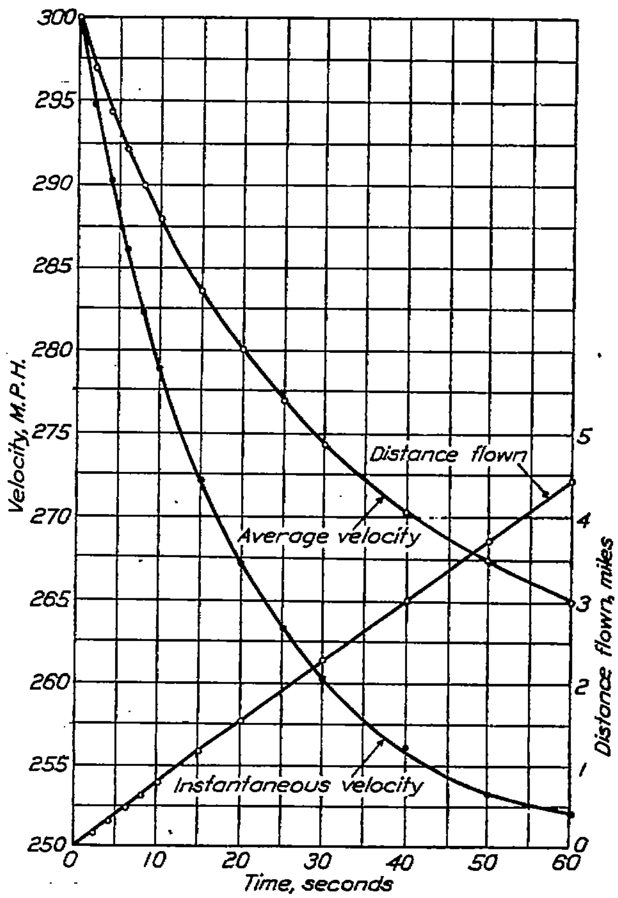

FIG. 5.—Initial velocity 300 M. P. H.

Effect of a diving start—
Normal velocity 250 M. P. H. W=2,100 lb. T=600 lb.

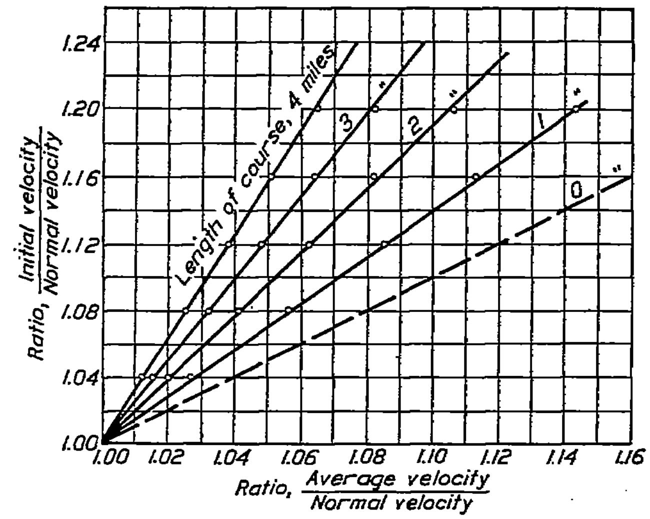

FIG. 6.—The effect of a diving start on average velocity. (High speed racing type airplanes.)